Activity 1.5: Using the Sunflower Family (Asteraceae) for Integrating Life Sciences and Mathematics

Background:

Spiral patterns of growth are characteristic of plants. Take a moment to think of spiral patterns you have observed in plants. Perhaps you thought of an artichoke, pinecone, or rose flower. Spiral patterns must confer success to plants; otherwise we could expect this pattern to be either rare or non-existent. The following discussion involves analyzing the growth patterns of inflorescences of the sunflower family (Asteraceae) to see how they conform to an interesting set of numbers, the Fibonacci numbers.

We teachers know that there are different ways to integrate subjects. For example, we could either start with plants and then include connections to mathematics, or go vice versa. Let's start with numbers.

Below, we have some initial numbers of the infinite set of Fibonacci numbers: 0, 1, 1, 2, 3, 5, 8, 13, 21, 34, 55, 89 ------.

The pattern for Fibonacci numbers is derived by summing the two anteceding numbers as demonstrated below:

0 + 1 = 1; 1 + 1 = 2; 1 + 2 = 3; 2 + 3 = 5; 3 + 5 = 8; 5 + 8 = 13; 8 + 13 = 21; 13 + 21 = 34; 21 + 34 = 55; 34 + 55 = 89; 55 + 89 = 144; etc.

The realm of Fibonacci numbers is vast and can relate to a variety of topics, allowing connections to multiple subjects and teaching across the curriculum. A useful reference to learn more about Fibonacci numbers is a book written by Garland (1987). As in other portions of this manual, we promote etymology for increasing word power both within and outside of science.

Although the Fibonacci set or series is infinite, the number of given plant parts constituting either parts of, or numbers of spirals are limited. In botany counting the items arranged in a spiral, such as bracts of an artichoke or the spirals of flowers in the head of a daisy can yield Fibonacci numbers. Usually there are clockwise and counter-clockwise spirals with consecutive Fibonacci numbers, such as 34 and 55. Let's look and see how pairs of Fibonacci numbers appear in three examples of the sunflower family.

We will start with a brief review of the inflorescence, the arrangement of flowers and attendant parts, in the sunflower family. The sunflower inflorescence is also called a head. The perimeter of a head consists of overlapping leaf-like **involucral bracts** (L. involucrum = wrapper, envelope < involvo = to wrap up, roll up; bractea = thin plate of metal, gold leaf, veneer). Bracts resemble sepals, often green in color. However, bracts are not parts of flowers; instead they are attendant distinct elements resembling sepals. They surround the flowers of a sunflower head. The flowers constituting the inflorescence have fused petals, the synsepalous condition. Flowers are typically of two types. Elongate bilaterally symmetrical **ray flowers** are located adjacent to the peripheral bracts. Due to their **corollas** (collective term for all the petals; L. corolla = diminutive of corona = crown) extending outward, they resemble petals of other flowering plants. Circular, radially symmetrical **disk flowers** fill in the remainder of the head. Both the ray and disk flowers are borne on a common **receptacle**. The **peduncle** is the stalk bearing the head or inflorescence.

Example #1: The Artichoke, Cynara scolymos (Gr. kynara = skolymos = artichoke)-How may we use an artichoke to exemplify a mathematical pattern? We will study the spiral arrangement of the involucral bracts. Recall that the bracts and the heart (= receptacle) are the tasty portions that we commonly garnish with mayonnaise. Consider consuming the boiled peduncle if you have not done so. You may find it to be a culinary delight. We can start by examining Figure 1. Note Figures 1-11 are in the same order in the accompanying PowerPoint. Do the bracts appear to be arranged into two spirals running in opposing directions? Are the spirals at the same or at different angles? Consider plucking off the bracts of an artichoke one at a time to observe further the spiral pattern. Now examine Figure 2 illustrating two sets of spirals running at different angles and in opposite directions. Is there an apparent relationship between the declination of the two sets of spirals to the number of spirals? Figure 3 is a top view of the artichoke. Notice how spiraling beautifully packs the bracts together. By spiraling, the bracts can be packed into less space. Is there a possible relationship between numbers 5 and 8 in the pattern of Fibonacci numbers? Yes, they are part of the Fibonacci sequence.

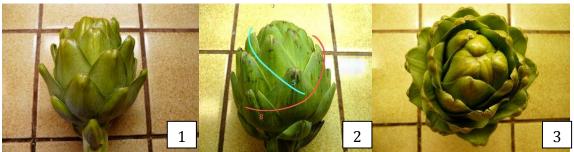


Figure 1. Involucral bracts spiral in on the head (inflorescence) of an artichoke. **Figure 2.** Involucral bracts of the inflorescence of an artichoke positioned as a steep series of spirals numbering 5 and a shallow series of spirals numbering 8. **Figure 3.** A top view of an artichoke shows the pattern of overlap of its bracts allowing for a compact arrangement of bracts.

An edible artichoke bears fine somewhat translucent elements borne on the receptacle (= heart). These elements are immature flowers. With maturation of a head, the flowers become large and their corollas appear blue or purple. The corollas are all of disc flowers. It may be worth stating that there are members of the sunflower family having only ray flowers constituting a head. A prime example is the dandelion (French dent de lion = tooth of the lion) also named *Taraxacum officinale* (New Latin taraxacum perhaps Persian tarashqun = dandelion; L. officinale = may be related to pharmaceutical use).

Example #2: The Head of an Ox-eye Daisy, *Leucanthemum vulgare* (Gr. leukos = white, bright, light; anthos = flower; L. vulgaris = common, usual) – The ox-eye Daisy is an exotic (Gr. exotikos = foreign, not native) plant native to Europe. It serves well to exemplify the arrangement of flowers in the head of a member of the sunflower family. **Figure 4** shows the head of an ox-eye daisy. White marginal ray flowers form a border resembling petals and surrounding the yellow central disk flowers. Notice in **Figure 5** the rim of open disk flowers surrounding the less mature inner disk flowers. We will focus on the spiral arrangement of the disk flowers shown in **Figures 6 and 7**. Look at **Figure 6**. Do you see the spiral with red dots, spiraling to the right and ending with a circle near the upper portion of the figure? Each number represents the ending of a spiral. Although

the red numbers are small, do you count 21 clockwise spirals? ----- Next, look at **Figure 7** with blue labeling and notice the spiral with blue dots spiraling to the left and ending with a circle near the upper portion of the figure. Each blue number represents the ending of a spiral. Note 34 spirals are labeled in blue. What are the shapes of the developing flowers? Do you notice six-sided objects, hexagons? Each flower is placed where a part of each of the two spirals cross at or about 90 degrees. Does six-sidedness allow for tight packing, and, therefore, little "wasted" space between flowers? ---- If the spirals were straightened out, would the cluster of flowers require more space? Think of the area used by spirals versus straightened strands.



Figure 4. The head of an ox-eye daisy viewed from above shows the white marginal ray flowers and the yellow central disk flowers.

Figure 5. The head of an ox-eye daisy viewed from above shows more closely the spiraling patterns of the yellow central disk flowers.



Figure 6. The spiral pattern revolving to the right displayed by the disk flowers of the oxeye daisy. The spiral with the red dots ending in a red circle represents one spiral. Each number is the ending of a spiral. Twenty-one spirals constitute the entire set.



Figure 7. The spiral pattern revolving to the left displayed by the disk flowers of the oxeye daisy. The spiral with the blue dots ending in a blue circle represents one spiral. Each number is the ending of a spiral. Thirty-four spirals constitute the entire set.

Example #3: The Head of a Sunflower, Helianthus annuus (Gr. helios = the sun; anthos = flower; L. annuus = annual, yearly) – We notice that both the ox-eye daisy and the sunflower share the same general pattern of ray and disk flower arrangement. A major difference is in the sizes of their heads. The sunflower head is the larger of the two. **Figure 8** displays a maturing sunflower head. As with the ox-eye daisy, notice the disk flowers are more mature near the ray flowers. Now look at **Figure 9**. Here we view the disk flowers up close. Like in the ox-eye daisy, maturation decreases toward the center. **Figure 10** shows a blue circle with a series of dots on developing flowers, representing one spiral of a group of flowers spiraling left and away from the center of the flower. Each blue number represents the ending of a spiral. How many numbers do you count of such spirals? Do you count 34? Next, look at **Figure 11.** To the left, a red circle can be seen with a series of red dots within a spiral running in the opposite direction and extending to the center. How many spirals are numbered? Do you count 55?

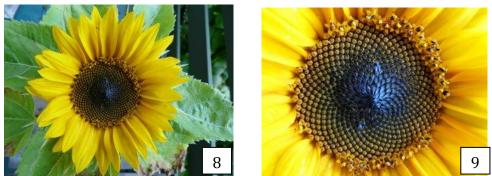


Figure 8. The maturing head of a sunflower. As with the ox-eye daisy, notice the disk flowers are more mature near the ray flowers.

Figure 9. The maturing head of a sunflower viewed up close to see the disk flowers. As with the ox-eye daisy, notice the disk flowers are more mature near the ray flowers.



Figure 10. Blue dots on developing flowers from the center extend outward, toward the blue circle in the upper left margin, representing a spiral. Each number is the ending of a spiral. Thirty-four spirals constitute the entire set.



Figure 11. Red dots on developing flowers from the center extend outward, toward the red circle in the left margin, representing a spiral. Each number is the ending of a spiral. Fifty-five spirals constitute the entire set.

Looking at the numbers of spirals that we have observed as we studied the artichoke (5/8), the ox-eye daisy (21/34), and the sunflower (34/55), would you say that they all belong to the Fibonacci series? What have we found regarding the Fibonacci pairs? Are they all of consecutive Fibonacci numbers? With the ox-eye daisy and sunflower, does there appear to be relationship between: 1) the size of an inflorescence and the numbers constituting a Fibonacci pair? 2) spiraling with more effective reduction in space needed for packing flowers? 3) flower shape in cross-section, either square, or hexagonal, as a way of reducing the amount of space needed for a given number of parts, either bracts, or flowers?

An extension of Fibonacci numbers worth considering involves spiral patterns of leaves. For example, if one were to compare the ratios of the number of leaves per number of revolutions to find leaves in longitudinal alignment, the ratio of the numbers consists of two Fibonacci numbers separated by one Fibonacci number, e.g, 3/5, 5/8, and 8/13. Like efficient use of space with spiraling in sunflower inflorescences, the aforementioned ratios allow for the most illumination of leaves, favoring photosynthesis. A good time to delve into this matter is once students have facility with fractions and decimals.

Objective/Goals:

Using science process skills (observing, analyzing, identifying, classifying) students will be able to identify patterns of growth in plants from the sunflower family and their relationship to Fibonacci numbers.

Time to complete:

Approximately 60 to 90 minutes

CA Science Content Standards

Fibonacci numbers are not covered in the Standards, yet are pervasive in nature.

Lesson Outline:

During Class:

- 1. Begin with a discussion about patterns in nature, with a focus on spirals.
- 2. Discuss patterns with numbers and present the Fibonacci sequence pattern.
- 3. Show photos or bring in artichokes and sunflowers for students to identify spiral patterns and Fibonacci numbers.

Preparation:

Artichokes and sunflowers need to be acquired before conducting this activity. An example of each could be used by 2-4 students.

Materials:

Artichoke, sunflower

Procedure:

Pre-Activity Discussion

- Begin class by asking students about patterns in nature they have observed.
- Show photos of different patterns in nature or take a walk around the school to observe patterns in flowers and leaves. In addition to the artichoke and sunflower, you may show photos of pinecones, agave, roses, or pineapples. Highlight spiral patterns through photos or during a walk.
- Discuss with students other patterns that can be observed with numbers (e.g., odd or even numbers, multiples of different numbers).
- Introduce the Fibonacci numbers (0, 1, 1, 2, 3, 5, 8....) and ask the students if they see any pattern. Explain to them the Fibonacci sequence, that adjacent numbers are summed to equal the next greater number. Next discuss the way some spiral patterns in nature are related to the Fibonacci numbers.

Hands-on Activity

- Using the PowerPoint slides, start with the photo of the artichoke, help the students to identify the clockwise and counterclockwise spirals. Have them count the number of spirals they see. Does the number of spirals represent Fibonacci numbers?
- Next using the photo of the sunflower, work with students to identify and count the spirals they see. Does the number of spirals represent Fibonacci numbers?
- Divide the class into groups of 2-4 students, depending on the number of flowers you have. An artichoke and a sunflower can be provided to each group.

 Next have students try to identify these spirals using the artichoke and sunflower samples. They may want to remove the bracts on the artichoke to help observe the pattern.

Group Discussion (answers are discussed in the Background section)

- After the students are familiar with the Fibonacci numbers and identifying the patterns in the artichoke and sunflower, discuss why the Fibonacci pattern may be useful in nature. Ask them if they observed the shape of the flowers (hexagons) or bracts and why this shape might be useful for the flower.
- Conclude the discussion by asking students about other spiral patterns they have observed in nature that might relate to the Fibonacci numbers.

Extension Activities:

1. What is the behavior of ratios of adjacent Fibonacci numbers? Students who understand ratios and fractions, could explore patterns of placing the lower number of the Fibonacci pair in the numerator:

1/1 = 1, 1/2 = 0.5, 2/3 = 0.6667, 3/5 = 0.6000, 5/8 = 0.6250, 8/13 = 0.6154, 13/21 = 0.6190, 21/34 = 0.6176

Or by placing the larger number of adjacent pairs in the numerator:

1/1 = 1; 2/1 = 2, 3/2 = 1.5, 5/3 = 1.666, 8/5 = 1.6, 13/8 = 1.625, 21/13 = 1.6153,--. Both of the two series have an interesting way of the ratios converging, but not becoming the same. The series of ratios are irrational.

Guide Resources:

Artichoke and Sunflower Photos Using the Sunflower Family (Asteraceae) for Integrating Life Sciences and Mathematics PowerPoint

References:

Adam, John A. 2009. <u>A Mathematical Nature Walk.</u> Princeton University Press, Princeton, NJ. We judge this book is too complex mathematically for elementary and middle school. However, it is an "eye opener" into the richness of places like "in the garden" and "in the neighborhood" were mathematical examples abound.

Bell, Adrian D. 2007. Plant Form: <u>An Illustrated Guide to Flowering Plant Morphology.</u> Timber Press, Portland, OR. This account is constructed for individuals with some background in plant morphology. However, we recommend considering the use of this text for photographs, figures, and discussions on given flowering plant parts.

Garland, Trudi Hammel. 1987. <u>Fascinating Fibonaccis Mystery and Magic in Numbers.</u> Dale Seymour Publications, Palo Alto, CA. This book contains information on history and application of Fibonacci numbers to nature, art and architecture, music and poetry, science and technology, etc. We judge the book to be a gem of ideas for "teaching across the curriculum."

Garland, Trudi Hammel. 1998. <u>Fibonacci Fun: Fascinating Activities with Intriguing</u> <u>Numbers.</u> Dale Seymour Publications, Palo Alto, CA. This book contains easy to use activities and projects for students and makes useful connections across the curriculum. Jaeger, Edmund C. 1978. <u>A Source-Book of Biological Names and Terms.</u> Charles C. Thomas Publisher, Springfield, IL. This book is out of press. It is a treasure trove for encouraging the learning of roots of scientific terms. Such learning can carry over into the rest of the curriculum.

Pappas, Theoni. 1991. <u>More Joy of Mathematics Exploring Mathematics All Around You.</u> Wide World Publishing/Tetra, San Carlos, CA. We put this book in the same category of Garland 1987 listed above.

Stevens. Peter S. 1974. <u>Patterns in Nature.</u> Atlantic Monthly Press Book, Boston, MA . This book contains a wealth of information on patterns and form. Although form may be restricted, nature has still brought "beauty and harmony to the natural world." The book is out of print, but available as used through Amazon.com.

Williams Jr., Ernest H. 2005. <u>The Nature Handbook A Guide to Observing the Great</u> <u>Outdoors.</u> Oxford University Press, NY. This account is rich in examples collectively dealing with size and shape, structural and behavioral adaptations, the diversity of life, etc.

Other sources: There is a wealth of information also on-line.

PowerPoint slides for Unit 1: Activity 1.5: Using the Sunflower Family (Asteraceae) for Integrating Life Sciences and Mathematics

The order of the slides follows Figure 1-11 in the Background



Figure 1. Involucral bracts spiral in on the head (inflorescence) of an artichoke.



Figure 2. Involucral bracts of the inflorescence of an artichoke positioned as a steep series of spirals numbering 5 and a shallow series of spirals numbering 8.



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